FUNCIONES HIPERBÓLICAS.

1.- Definición de las funciones hiperbólicas.

$\operatorname{senh} x = \dfrac{e^x - e^{-x}}{2}$ $\qquad \cosh x = \dfrac{e^x + e^{-x}}{2}$ $\qquad \tanh x = \dfrac{e^x - e^{-x}}{e^x + e^{-x}} = \dfrac{e^{2x} - 1}{e^{2x} + 1}$

2.- Funciones recíprocas.

$\operatorname{csch} x = \dfrac{1}{\operatorname{senh} x}$ $\qquad \operatorname{sech} x = \dfrac{1}{\cosh x}$ $\qquad \coth x = \dfrac{1}{\tanh x}$

$\operatorname{csch} x = \dfrac{2}{e^x - e^{-x}}$ $\qquad \operatorname{sech} x = \dfrac{2}{e^x + e^{-x}}$ $\qquad \coth x = \dfrac{e^x + e^{-x}}{e^x - e^{-x}} = \dfrac{e^{2x} + 1}{e^{2x} - 1}$

3.- Identidades hiperbólicas fundamentales.

$\operatorname{senh} x \operatorname{csch} x = 1$ $\qquad \cosh x \operatorname{sech} x = 1$ $\qquad \tanh x \coth x = 1$ $\qquad \tanh x = \dfrac{\operatorname{senh} x}{\cosh x}$ $\qquad \coth x = \dfrac{\cosh x}{\operatorname{senh} x}$

4.- Identidades pitagóricas.

$\cosh^2 x - \operatorname{senh}^2 x = 1$ $\qquad \coth^2 x - 1 = \operatorname{csch}^2 x$ $\qquad 1 - \tanh^2 x = \operatorname{sech}^2 x$

5.- Otras identidades.

$\cosh x + \operatorname{senh} x = e^x$ $\qquad \cosh x - \operatorname{senh} x = e^{-x}$

6.- Equivalencia entre las funciones hiperbólicas.

	$\operatorname{senh} x$	$\cosh x$	$\tanh x$	$\operatorname{csch} x$	$\operatorname{sech} x$	$\coth x$
$\operatorname{senh} x$	1	$\sqrt{\cosh^2 x - 1}$	$\dfrac{\tanh x}{\sqrt{1-\tanh^2 x}}$	$\dfrac{1}{\operatorname{csch} x}$	$\dfrac{\sqrt{1-\operatorname{sech}^2 x}}{\operatorname{sech} x}$	$\dfrac{1}{\sqrt{\coth^2 x - 1}}$
$\cosh x$	$\sqrt{1+\operatorname{senh}^2 x}$	1	$\dfrac{1}{\sqrt{1-\tanh^2 x}}$	$\dfrac{\sqrt{1+\operatorname{csch}^2 x}}{\operatorname{csch} x}$	$\dfrac{1}{\operatorname{sech} x}$	$\dfrac{\coth x}{\sqrt{\coth^2 x - 1}}$
$\tanh x$	$\dfrac{\operatorname{senh} x}{\sqrt{1+\operatorname{senh}^2 x}}$	$\dfrac{\sqrt{\cosh^2 x - 1}}{\cosh x}$	1	$\dfrac{1}{\sqrt{1+\operatorname{csch}^2 x}}$	$\sqrt{1-\operatorname{sech}^2 x}$	$\dfrac{1}{\coth x}$
$\operatorname{csch} x$	$\dfrac{1}{\operatorname{senh} x}$	$\dfrac{1}{\sqrt{\cosh^2 x - 1}}$	$\dfrac{\sqrt{1-\tanh^2 x}}{\tanh x}$	1	$\dfrac{\operatorname{sech} x}{\sqrt{1-\operatorname{sech}^2 x}}$	$\sqrt{\coth^2 x - 1}$
$\operatorname{sech} x$	$\dfrac{1}{\sqrt{1+\operatorname{senh}^2 x}}$	$\dfrac{1}{\cosh x}$	$\sqrt{1-\tanh^2 x}$	$\dfrac{\operatorname{csch} x}{\sqrt{1+\operatorname{csch}^2 x}}$	1	$\dfrac{\sqrt{\coth^2 x - 1}}{\coth x}$
$\coth x$	$\dfrac{\sqrt{1-\operatorname{senh}^2 x}}{\operatorname{senh} x}$	$\dfrac{\cosh x}{\sqrt{\cosh^2 x - 1}}$	$\dfrac{1}{\tanh x}$	$\sqrt{1+\operatorname{csch}^2 x}$	$\dfrac{1}{\sqrt{1-\operatorname{sech}^2 x}}$	1

7.- Argumento doble.

$\operatorname{senh} 2x = 2 \operatorname{senh} x \cosh x$ $\qquad \cosh 2x = \cosh^2 x + \operatorname{senh}^2 x$

$\cosh 2x = 2\operatorname{senh}^2 x + 1$ $\qquad \cosh 2x = 2\cosh^2 x - 1$ $\qquad \tanh 2x = \dfrac{2\tanh x}{1 + \tanh^2 x}$

8.- Argumento triple.

$\operatorname{senh} 3x = 4\operatorname{senh}^3 x + 3\operatorname{senh} x$ $\qquad \cosh 3x = 4\cosh^3 x - 3\cosh x$

$\tanh 3x = \dfrac{3\tanh x + \tanh^3 x}{1 + 3\tanh^2 x}$

9.- Argumento mitad.

$\operatorname{senh}\left(\dfrac{x}{2}\right) = \sqrt{\dfrac{\cosh x - 1}{2}}$ $\qquad \cosh\left(\dfrac{x}{2}\right) = \sqrt{\dfrac{\cosh x + 1}{2}}$ $\qquad \tanh\left(\dfrac{x}{2}\right) = \sqrt{\dfrac{\cosh x - 1}{\cosh x + 1}}$

$\tanh\left(\dfrac{x}{2}\right) = \dfrac{\operatorname{senh} x}{\cosh x + 1}$ $\qquad \tanh\left(\dfrac{x}{2}\right) = \dfrac{\cosh x - 1}{\operatorname{senh} x}$ $\qquad \tanh\left(\dfrac{x}{2}\right) = \coth x - \operatorname{csch} x$

$\coth\left(\dfrac{x}{2}\right) = \sqrt{\dfrac{\cosh x + 1}{\cosh x - 1}}$ $\qquad \coth\left(\dfrac{x}{2}\right) = \dfrac{\operatorname{senh} x}{\cosh x - 1}$ $\qquad \coth\left(\dfrac{x}{2}\right) = \dfrac{\cosh x + 1}{\operatorname{senh} x}$

$\coth\left(\dfrac{x}{2}\right) = \coth x + \operatorname{csch} x$ $\qquad \operatorname{senh}^2 x = \tfrac{1}{2}(\cosh 2x - 1)$ $\qquad \cosh^2 x = \tfrac{1}{2}(\cosh 2x + 1)$

10.- Suma y diferencia de dos argumentos.

$\operatorname{senh}(x \pm y) = \operatorname{senh} x \cosh y \pm \cosh x \operatorname{senh} y$ $\qquad \cosh(x \pm y) = \cosh x \cosh y \pm \operatorname{senh} x \operatorname{senh} y$

11.- Fórmulas de reducción.

$\operatorname{senh}(-x) = -\operatorname{senh} x$ $\qquad \operatorname{csch}(-x) = -\operatorname{csch} x$ $\qquad \cosh(-x) = \cosh x$

$\operatorname{sech}(-x) = \operatorname{sech} x$ $\qquad \tanh(-x) = -\tanh x$ $\qquad \coth(-x) = -\coth x$

12.- Suma y diferencia de funciones hiperbólicas.

$\operatorname{senh} x + \operatorname{senh} y = 2\operatorname{senh}\left(\dfrac{x+y}{2}\right)\cosh\left(\dfrac{x-y}{2}\right)$ $\qquad \operatorname{senh} x - \operatorname{senh} y = 2\operatorname{senh}\left(\dfrac{x-y}{2}\right)\cosh\left(\dfrac{x+y}{2}\right)$

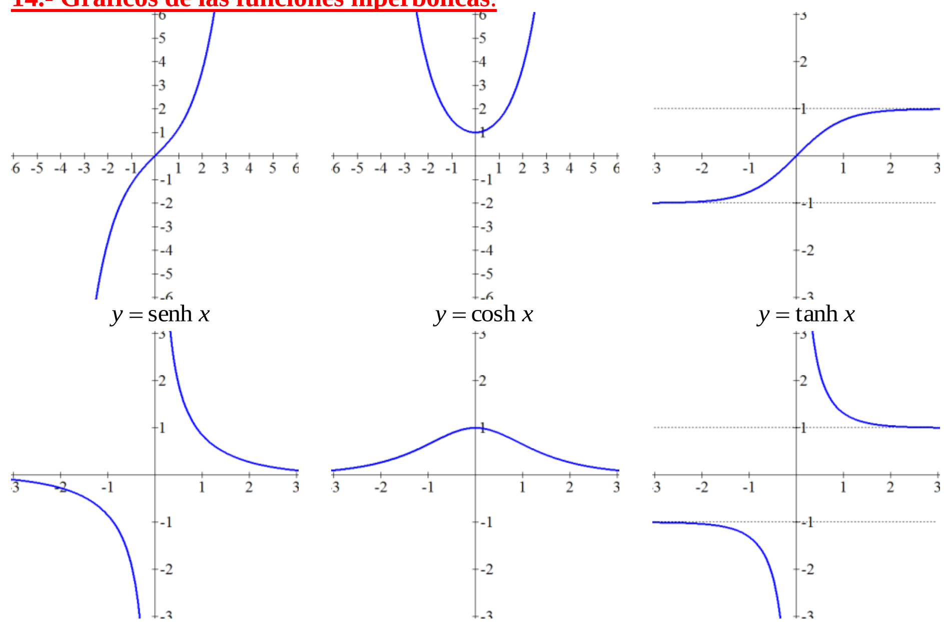
$\cosh x + \cosh y = 2\cosh\left(\dfrac{x+y}{2}\right)\cosh\left(\dfrac{x-y}{2}\right)$ $\qquad \cosh x - \cosh y = 2\operatorname{senh}\left(\dfrac{x+y}{2}\right)\operatorname{senh}\left(\dfrac{x-y}{2}\right)$

13.- Producto.

$\operatorname{senh} x \operatorname{senh} y = \tfrac{1}{2}[\cosh(x+y) - \cosh(x-y)]$ $\qquad \cosh x \cosh y = \tfrac{1}{2}[\cosh(x+y) + \cosh(x-y)]$

$\operatorname{senh} x \cosh y = \tfrac{1}{2}[\operatorname{senh}(x+y) - \operatorname{senh}(x-y)]$

14.- Gráficos de las funciones hiperbólicas.



$y = \operatorname{senh} x$ $\qquad y = \cosh x$ $\qquad y = \tanh x$

$y = \operatorname{csch} x$ $\qquad y = \operatorname{sech} x$ $\qquad y = \coth x$

15.- Derivadas de las funciones hiperbólicas.

$\dfrac{d}{dx}(\operatorname{senh} u) = \cosh u \dfrac{du}{dx}$ $\qquad \dfrac{d}{dx}(\cosh u) = \operatorname{senh} u \dfrac{du}{dx}$ $\qquad \dfrac{d}{dx}(\tanh u) = \operatorname{sech}^2 u \dfrac{du}{dx}$

$\dfrac{d}{dx}(\operatorname{csch} u) = -\operatorname{csch} u \coth u \dfrac{du}{dx}$ $\qquad \dfrac{d}{dx}(\operatorname{sech} u) = -\operatorname{sech} u \tanh u \dfrac{du}{dx}$ $\qquad \dfrac{d}{dx}(\coth u) = -\operatorname{csch}^2 u \dfrac{du}{dx}$

16.- Integrales de las funciones hiperbólicas.

$\int \operatorname{senh} u \, du = \cosh u + C$ $\qquad \int \cosh u \, du = \operatorname{senh} u + C$ $\qquad \int \tanh u \, du = -\ln(\operatorname{sech} u) + C$

$\int \operatorname{csch} u \, du = \ln[\tanh(\tfrac{1}{2}u)] + C$ $\qquad \int \operatorname{sech} u \, du = \tan^{-1}(\operatorname{senh} u) + C$ $\qquad \int \coth u \, du = \ln(\operatorname{senh} u) + C$

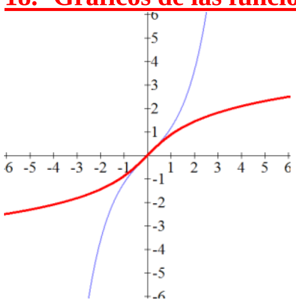
$\int \operatorname{sech}^2 u \, du = \tanh u + C$ $\qquad \int \operatorname{csch}^2 u \, du = -\coth u + C$ $\qquad \int \operatorname{sech} u \tanh u \, du = -\operatorname{sech} u + C$

$\int \operatorname{csch} u \coth u \, du = -\operatorname{csch} u + C$

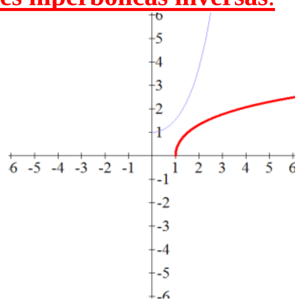
17.- Definición de las funciones hiperbólicas inversas.

	Definición	Domino	Rango
Seno hiperbólico inverso	$\operatorname{senh}^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$	R	R
Coseno hiperbólico inverso	$\cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right)$	[1 , ∝)	[0 , ∝)
Tangente hiperbólica inversa	$\tanh^{-1} x = \ln\sqrt{\dfrac{1+x}{1-x}}$ $\tanh^{-1} x = \tfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)$	(-1 , 1)	R
Cosecante hiperbólica inversa	$\operatorname{csch}^{-1} x = \ln\left(\dfrac{1+\sqrt{1+x^2}}{x}\right)$ $\operatorname{csch}^{-1} x = \ln\left(\dfrac{1}{x} + \sqrt{\dfrac{1+x^2}{x^2}}\right)$	(-∝ , 0) ∪ (0 , ∝)	(-∝ , 0) ∪ (0 , ∝)
Secante hiperbólica inversa	$\operatorname{sech}^{-1} x = \ln\left(\dfrac{1+\sqrt{1-x^2}}{x}\right)$ $\operatorname{sech}^{-1} x = \ln\left(\dfrac{1}{x} + \sqrt{\dfrac{1-x^2}{x^2}}\right)$	(0 , 1]	[0 , ∝)
Cotangente hiperbólica inversa	$\coth^{-1} x = \ln\sqrt{\dfrac{x+1}{x-1}}$ $\coth^{-1} x = \tfrac{1}{2}\ln\left(\dfrac{x+1}{x-1}\right)$	(-∝ , -1) ∪ (1 , ∝)	(-∝ , 0) ∪ (0 , ∝)

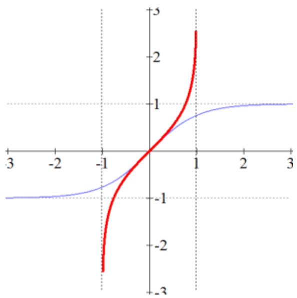
18.- Gráficos de las funciones hiperbólicas inversas.



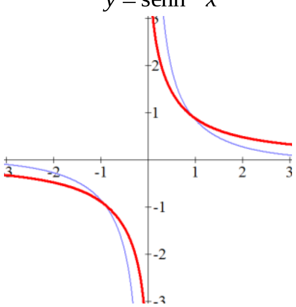
$y = \operatorname{senh}^{-1} x$



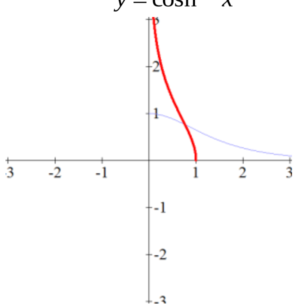
$y = \cosh^{-1} x$



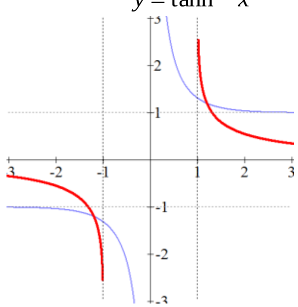
$y = \tanh^{-1} x$



$y = \operatorname{csch}^{-1} x$



$y = \operatorname{sech}^{-1} x$



$y = \coth^{-1} x$

27.- Identidades hiperbólicas inversas.

	$\operatorname{senh}^{-1}\theta$	$\cosh^{-1}\theta$	$\tanh^{-1}\theta$	$\operatorname{csch}^{-1}\theta$	$\operatorname{sech}^{-1}\theta$	$\coth^{-1}\theta$
$\operatorname{senh}^{-1}\theta$	1	$\cosh^{-1}\sqrt{1+\theta^2}$	$\tanh^{-1}\dfrac{\theta}{\sqrt{1+\theta^2}}$	$\operatorname{csch}^{-1}\dfrac{1}{\theta}$	$\operatorname{sech}^{-1}\dfrac{1}{\sqrt{1+\theta^2}}$	$\coth^{-1}\dfrac{\sqrt{1+\theta^2}}{\theta}$
$\cosh^{-1}\theta$	$\operatorname{senh}^{-1}\sqrt{\theta^2-1}$	1	$\tanh^{-1}\dfrac{\sqrt{\theta^2-1}}{\theta}$	$\operatorname{csch}^{-1}\dfrac{1}{\sqrt{\theta^2-1}}$	$\operatorname{sech}^{-1}\dfrac{1}{\theta}$	$\coth^{-1}\dfrac{\theta}{\sqrt{\theta^2-1}}$
$\tanh^{-1}\theta$	$\operatorname{senh}^{-1}\dfrac{\theta}{\sqrt{1-\theta^2}}$	$\cosh^{-1}\dfrac{1}{\sqrt{1-\theta^2}}$	1	$\operatorname{csch}^{-1}\dfrac{\sqrt{1-\theta^2}}{\theta}$	$\operatorname{sech}^{-1}\sqrt{1-\theta^2}$	$\coth^{-1}\dfrac{1}{\theta}$
$\operatorname{csch}^{-1}\theta$	$\operatorname{senh}^{-1}\dfrac{1}{\theta}$	$\cosh^{-1}\dfrac{\sqrt{\theta^2+1}}{\theta}$	$\tanh^{-1}\dfrac{1}{\sqrt{\theta^2+1}}$	1	$\operatorname{sech}^{-1}\dfrac{\theta}{\sqrt{\theta^2+1}}$	$\coth^{-1}\sqrt{\theta^2+1}$
$\operatorname{sech}^{-1}\theta$	$\operatorname{senh}^{-1}\dfrac{\sqrt{1-\theta^2}}{\theta}$	$\cosh^{-1}\dfrac{1}{\theta}$	$\tanh^{-1}\sqrt{1-\theta^2}$	$\operatorname{csch}^{-1}\dfrac{\theta}{\sqrt{1-\theta^2}}$	1	$\coth^{-1}\dfrac{1}{\sqrt{1-\theta^2}}$
$\coth^{-1}\theta$	$\operatorname{senh}^{-1}\dfrac{1}{\sqrt{\theta^2-1}}$	$\cosh^{-1}\dfrac{\theta}{\sqrt{\theta^2-1}}$	$\tanh^{-1}\dfrac{1}{\theta}$	$\operatorname{csch}^{-1}\sqrt{\theta^2-1}$	$\operatorname{sech}^{-1}\dfrac{\sqrt{\theta^2-1}}{\theta}$	1

19.- Derivadas de las funciones hiperbólicas inversas.

$$\frac{d}{dx}(\operatorname{senh}^{-1}u) = \frac{1}{\sqrt{u^2+1}}\frac{du}{dx} \qquad \frac{d}{dx}(\cosh^{-1}u) = \frac{1}{\sqrt{u^2-1}}\frac{du}{dx} \qquad \frac{d}{dx}(\tanh^{-1}u) = \frac{1}{1-u^2}\frac{du}{dx}$$

$$\frac{d}{dx}(\operatorname{csch}^{-1}u) = \frac{-1}{|u|\sqrt{1+u^2}}\frac{du}{dx} \qquad \frac{d}{dx}(\operatorname{sech}^{-1}u) = \frac{-1}{|u|\sqrt{1-u^2}}\frac{du}{dx} \qquad \frac{d}{dx}(\coth^{-1}u) = \frac{1}{1-u^2}\frac{du}{dx}$$

20.- Integrales de las funciones hiperbólicas inversas.

$$\int \operatorname{senh}^{-1}x\,dx = x\operatorname{senh}^{-1}x - \sqrt{1+x^2} + c \qquad \int \cosh^{-1}x\,dx = x\cosh^{-1}x - \sqrt{x^2-1} + c$$

21.- Integrales cuyas primitivas son funciones hiperbólicas inversas.

$$\int \frac{du}{\sqrt{u^2+a^2}} = \ln\left(u+\sqrt{u^2+a^2}\right)+C \qquad \int \frac{du}{\sqrt{u^2+a^2}} = \operatorname{senh}^{-1}\frac{u}{a}+C$$

$$\int \frac{du}{\sqrt{u^2-a^2}} = \ln\left(u+\sqrt{u^2-a^2}\right)+C \qquad \int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}\frac{u}{a}+C$$

$$\int \frac{du}{a^2-u^2} = \frac{1}{2a}\ln\left(\frac{a+u}{a-u}\right)+C \qquad \int \frac{du}{a^2-u^2} = \frac{1}{a}\tanh^{-1}\frac{u}{a}+C$$

$$\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left(\frac{a-u}{a+u}\right)+C \qquad \int \frac{du}{a^2-u^2} = \frac{1}{a}\coth^{-1}\frac{u}{a}+C$$

$$\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\ln\left(\frac{a+\sqrt{a^2+u^2}}{u}\right)+C \qquad \int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\operatorname{csch}^{-1}\frac{u}{a}+C$$

$$\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\ln\left(\frac{a+\sqrt{a^2-u^2}}{u}\right)+C \qquad \int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\operatorname{sech}^{-1}\frac{u}{a}+C$$

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